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A CONSERVATIVE BOX-SCHEME FOR THE EULER EQUATIONS

JEAN-JACQUES CHATTOT*
Department of Mechanical and Aeronautical Engineering, University of California at Davis, Davis, CA 95616, USA

SUMMARY

The work presented in this paper shows that the mixed-type scheme of Murman and Cole, originally developed for a scalar equation, can be extended to systems of conservation laws. A characteristic scheme for the equations of gas dynamics is introduced that has a close connection to a four operator scheme for the Burgers–Hopf equation. The results indicate that the scheme performs well on the classical test cases. The scheme has no tuning parameters and can be interpreted as the projection of an L_∞-stable scheme. At steady state second order accuracy is obtained as a by-product of the box-scheme feature.

KEY WORDS: conservative box-scheme; Euler equations; gas dynamics

1. INTRODUCTION

The pioneering work of Murman and Cole on the solution by relaxation of the transonic small disturbance equation (TSD) [1] completed by the sequel paper of Murman [2], was a breakthrough in many ways, and foremost in computing efficiency. However, it may not have been said enough that the method had two other features that are highly desirable and not commonly found in schemes for the Euler equations: there is no tuning parameter to adjust, and the method is remarkably robust.

In order to prepare the ground for the Euler equations, the link between the mixed-scheme for the 1-D TSD potential equation and a corresponding scheme for Burgers (inviscid) equation is now established. The governing equation for the small disturbance potential for flows near Mach one can be written as:

$$-\frac{\partial}{\partial x}\left[\frac{1}{2}\left(\frac{\partial \varphi}{\partial x}\right)^2\right]=0$$

+ boundary conditions.

The perturbation potential φ is defined at the nodes and two switches are defined to characterize the flow regime at each point:

$$u_b=\frac{\varphi_i-\varphi_{i-2}}{2\Delta x}, \qquad u_c=\frac{\varphi_{i+1}-\varphi_{i-1}}{2\Delta x}.$$

* Correspondence to: Department of Mechanical and Aeronautical Engineering, University of California at Davis, Davis, CA 95616, USA.

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Consider now the Burgers–Hopf (B-H) equation for the disturbance velocity:

$$\frac{\partial u}{\partial t}+\frac{\partial}{\partial x}\left[\frac{u^2}{2}\right]=0$$

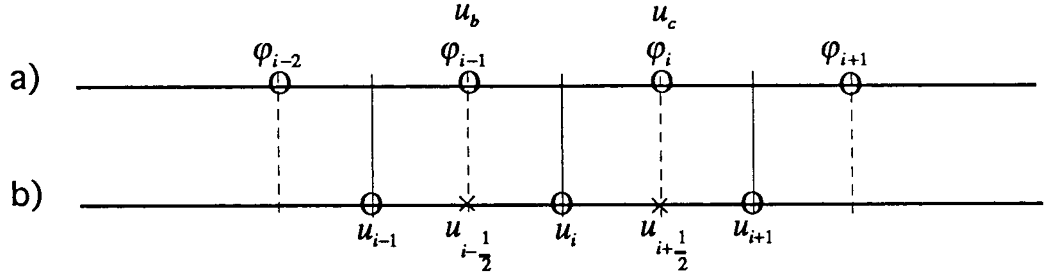
+ initial and boundary conditions. If the following discrete relation is introduced:

$$u_i=\frac{\varphi_i-\varphi_{i-1}}{\Delta x},$$

the switches can be rewritten in terms of u as:

$$u_{i-1/2}=\frac{u_{i-1}+u_i}{2}=\frac{\varphi_i-\varphi_{i-2}}{2\Delta x}, \qquad u_{i+1/2}=\frac{u_i+u_{i+1}}{2}=\frac{\varphi_{i+1}-\varphi_{i-1}}{2\Delta x}.$$

The correspondence between the two formulations is best depicted on the following sketch, with (a) defining the potential and the switches, and (b) the corresponding quantities for the velocity formulation:



This indicates that the four operators mixed scheme of Murman [2] has an exact equivalent for the B-H equation. However, a slightly different scheme will be introduced, with a different sonic point operator, that eliminates expansion shocks.

2. A MIXED-SCHEME FOR THE BURGERS–HOPF EQUATION

Consider the following mixed scheme:

(i) $u_{i-1/2}>0 \quad u_{i+1/2}>0$

$$\frac{u_i^{n+1}-u_i^n}{\Delta t}+\frac{(u_i^n)^2/2-(u_{i-1}^n)^2/2}{\Delta x}=0, \qquad \Delta t\le\frac{\Delta x}{u_{i-1/2}}$$

(ii) $u_{i-1/2}>0 \quad u_{i+1/2}<0$

$$\frac{u_i^{n+1}-u_i^n}{\Delta t}+\frac{(u_{i+1}^n)^2/2-(u_i^n)^2/2}{\Delta x}+\frac{(u_i^n)^2/2-(u_{i-1}^n)^2/2}{\Delta x}=0, \qquad \Delta t\le\frac{\Delta x}{u_{i-1/2}-u_{i+1/2}}$$

(iii) $u_{i-1/2}<0 \quad u_{i+1/2}>0$

$$\frac{u_i^{n+1}-u_i^n}{\Delta t}+\frac{u_i^n\dfrac{u_{i+1}^n-u_{i-1}^n}{2\Delta x}}{1+\dfrac{\Delta t}{\Delta x}(u_{i+1/2}^n-u_{i-1/2}^n)}=0, \qquad \Delta t<\infty$$

(iv) $u_{i-1/2}<0 \quad u_{i+1/2}<0$

$$\frac{u_i^{n+1}-u_i^n}{\Delta t}+\frac{(u_{i+1}^n)^2/2-(u_i^n)^2/2}{\Delta x}=0, \qquad \Delta t \leq \frac{\Delta x}{-u_{i+1/2}}$$

It can be noted that the sonic point, scheme (iii), is not a conservative discretization and it can be shown that in the transient the conservation error is proportional to Δx. However, it is conservative at steady state. It is the price paid for the elimination of expansion shocks. This scheme is locally implicit and, as such, has no time step requirement, as it can be written:

$$\frac{u_i^{n+1}-u_i^n}{\Delta t}+u_i^{n+1}\frac{u_{i+1}^n-u_{i-1}^n}{2\,\Delta x}=0$$

The implicit character is achieved through the coefficient, not the space derivative. This brings unconditional stability for the sonic point.

Some comments need to be made regarding stability. Since the governing equation is non-linear, the study of stability has been carried out after linearization, using Von Neumann method. The linearization for the supersonic point operator (i) and subsonic point operator (iv) is straightforward, with characteristic speeds $u_{i+1/2}$ and $u_{i-1/2}$ taken as constant.

At the sonic point, the implicitness is recognized and the space derivative $(u_{i+1}^n-u_{i-1}^n)/2\,\Delta x$ is considered merely as a positive constant. For the shock point, scheme (ii), the linearization involves two constant wave speeds with opposite signs, as:

$$\frac{(u_{i+1}^n)^2/2-(u_{i-1}^n)^2/2}{\Delta x}=u_{i+1/2}^n\frac{u_{i+1}^n-u_i^n}{\Delta x}+u_{i-1/2}^n\frac{u_i^n-u_{i-1}^n}{\Delta x}.$$

This analysis provides the above stability conditions, which have been found reliable in numerical tests [3]. In updating form, the mixed-scheme reads as follows:

$$\text{(i)}\ \ u_i^{n+1}=u_i^n-\frac{\Delta t}{\Delta x}u_{i-1/2}^n(u_i^n-u_{i-1}^n)$$

$$\text{(ii)}\ \ u_i^{n+1}=u_i^n-\frac{\Delta t}{\Delta x}u_{i+1/2}^n(u_{i+1}^n-u_i^n)-\frac{\Delta t}{\Delta x}u_{i-1/2}^n(u_i^n-u_{i-1}^n)$$

$$\text{(iii)}\ \ u_i^{n+1}=u_i^n-\frac{\dfrac{\Delta t}{2\,\Delta x}u_i^n(u_{i+1}^n-u_{i-1}^n)}{1+\dfrac{\Delta t}{\Delta x}(u_{i+1/2}^n-u_{i-1/2}^n)}$$

$$\text{(iv)}\ \ u_i^{n+1}=u_i^n-\frac{\Delta t}{\Delta x}u_{i+1/2}^n(u_{i+1}^n-u_i^n)$$

It can be shown that this scheme is second order accurate at steady state for the B-H equation with a source term modeling a quasi-one-dimensional flow in a slender nozzle [4].

Upon multiplying each equation in the scheme by $(u_i^{n+1}+u_i^n)/2$ and after some manipulation, the following expressions are obtained:

$$\text{(i)}\ \ -\frac{(u_i^{n+1})^2/2-(u_i^n)^2/2}{\Delta t}-u_{i-1/2}^n\frac{(u_i^n)^2/2-(u_{i-1}^n)^2/2}{\Delta x}=\frac{u_{i-1/2}^n}{2}\left(1-\frac{\Delta t}{\Delta x}u_{i-1/2}^n\right)\frac{(u_i^n-u_{i-1}^n)^2}{\Delta x}$$

$$\text{(ii)}\ \ -\frac{(u_i^{n+1})^2/2-(u_i^n)^2/2}{\Delta t}-(u_{i+1/2}^n+u_{i-1/2}^n)\frac{(u_{i+1}^2)^2/2-(u_{i-1}^2)^2/2}{\Delta x}$$

$$=\frac{(u_{i-1/2}^n-u_{i+1/2}^n)}{\Delta x}\left(1-\frac{\Delta t}{\Delta x}(u_{i-1/2}^n+u_{i+1/2}^n)\right)\frac{(u_{i+1}^n+u_{i-1}^n)^2}{2}$$

$$\text{(iii)}\quad -\frac{(u_i^{n+1})^2/2-(u_i^n)^2/2}{\Delta t}-\left[\frac{u_i^n}{1+\dfrac{\Delta t}{\Delta x}(u_{i+1/2}^n-u_{i-1/2}^n)}\right]^2\frac{u_{i+1}^n-u_{i-1}^n}{2\,\Delta x}$$

$$=\frac{\Delta t}{2}\left[\frac{u_i^n\,\dfrac{u_{i+1}^n-u_{i-1}^n}{2\,\Delta x}}{1+\dfrac{\Delta t}{\Delta x}(u_{i+1/2}^n-u_{i-1/2}^n)}\right]^2$$

$$\text{(iv)}\quad -\frac{(u_i^{n+1})^2/2-(u_i^n)^2/2}{\Delta t}-u_{i+1/2}^n\,\frac{(u_{i+1}^n)^2/2-(u_i^n)^2/2}{\Delta x}$$

$$=-\frac{u_{i+1/2}^n}{2}\left(1+\frac{\Delta t}{\Delta x}\,u_{i+1/2}^n\right)\frac{(u_{i+1}^n-u_i^n)^2}{\Delta x}$$

Noting that the left-hand-sides are discrete representations of $\partial s/\partial t+u\,\partial s/\partial x$, where the 'entropy' is defined as $s=-u^2/2$, and that the right-hand-sides are all positive with the definition of the switches, these indicate that the entropy increases along the characteristics. At steady state the right-hand-sides vanish identically.

3. 1-D EULER EQUATIONS

The 1-D Euler equations can be written as:

$$\frac{\partial w}{\partial t}+\frac{\partial f(w)}{\partial x}=0,$$

where

$$w=\begin{pmatrix}\rho\\ \rho u\\ \rho E\end{pmatrix},\qquad f(w)=\begin{pmatrix}\rho u\\ \rho u^2+p\\ \rho uH\end{pmatrix},\qquad E=\frac{p}{(\gamma-1)\rho}+\frac{u^2}{2},\qquad H=\frac{\gamma p}{(\gamma-1)\rho}+\frac{u^2}{2}.$$

Let $A=\partial f/\partial w$ be the Jacobian matrix and $a^2=\gamma p/\rho$ the square of the speed of sound. The characteristic matrix C associated with the system of the 1-D Euler equations has three distinct eigenvalues:

$$|C|=|A-\lambda I|=0\qquad \lambda^{(j)}=\{u-a;\,u;\,u+a\},\quad j=1,2,3.$$

The system is totally hyperbolic. The left eigenvectors are obtained from:

$$l^{(j)}\cdot A=\lambda^{(j)}l^{(j)},\quad j=1,2,3.$$

The compatibility relations are:

$$CR^{(j)}\!:\ l^{(j)}\cdot\left(\frac{\partial w}{\partial t}+\frac{\partial f}{\partial x}\right)=0,\qquad j=1,2,3.$$

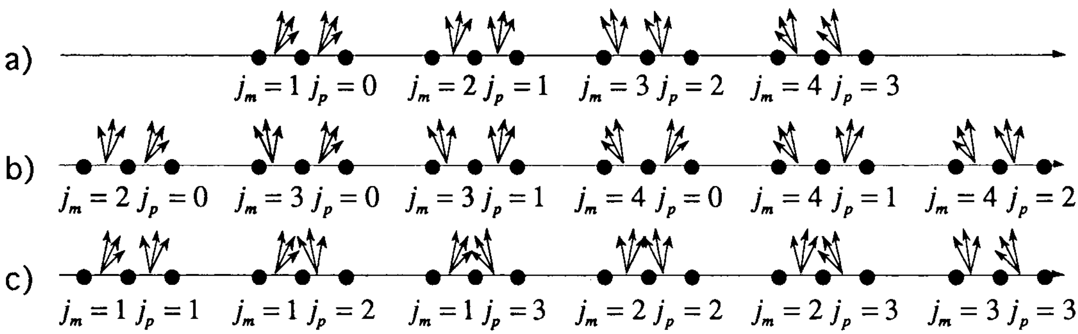
These represent interior derivatives along the characteristic lines defined by:

$$\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)_{C^{(j)}}=\lambda^{(j)},\quad j=1,2,3.$$

The three compatibility relations are equivalent to the original system. They form the basis of the numerical scheme described below.

4. A CHARACTERISTIC BOX-SCHEME

Consider the two boxes $[i-1, i]$, $[i, i+1]$ surrounding node i. The unknowns w_i^n are located at the nodes. In each box, the discrete jacobian matrix, eigenvalues and left eigenvectors are evaluated using Roe averages [5]. Depending on the signs of the characteristic speeds $\lambda^{(j)}_{i-1/2}$ and $\lambda^{(j)}_{i+1/2}$ several situations occur. Let j_m and j_p be two indices, such that j_m is the j-index of the first characteristic reaching point i from the left box, and j_p is the j-index of the last characteristic reaching point i from the right box. With the following values, $j_m = 1, 2, 3$ or 4, the value $j_m = 4$ corresponding to no characteristic reaching point i from the left box, and $j_p = 0, 1, 2$ or 3, $j_p = 0$ corresponding to no characteristic reaching point i from the right box. The total number of characteristics running into point i is $n_{\text{char}} = 4 - j_m + j_p$. The many different situations reduce to the three following cases: point i is either a regular point if $n_{\text{char}} = 3$, a 'sonic' point if $n_{\text{char}} < 3$ or a 'shock' point if $n_{\text{char}} > 3$. All the cases for the three situations are depicted in the sketch below.



Thus, only three if statements are needed for the coding of the scheme, as would also be the case in multiple dimensions.

At a regular point, the three compatibility relations are used and, for example in case (a)-2 corresponding to $u^n_{i-1/2} - a^n_{i-1/2} < 0 < u^n_{i-1/2} \quad - u^n_{i+1/2} - a^n_{i+1/2} < 0 < u^n_{i+1/2}$, they read:

$$\begin{bmatrix} 0 \\ l^{(2)}_{i-1/2} \\ l^{(3)}_{i-1/2} \end{bmatrix} \cdot \left(\frac{w_i^{n+1} - w_i^n}{\Delta t} + \frac{f_i^n - f_{i-1}^n}{\Delta x} \right) + \begin{bmatrix} l^{(1)}_{i+1/2} \\ 0 \\ 0 \end{bmatrix} \cdot \left(\frac{w_i^{n+1} - w_i^n}{\Delta t} + \frac{f_{i+1}^n - f_i^n}{\Delta x} \right) = 0.$$

At a 'sonic' point, one or more compatibility relations are missing. One or more new equations are introduced based on the exact solution for an expansion fan. For example, in case (b)-1 corresponding to $u^n_{i-1/2} - a^n_{i-1/2} < 0 < u^n_{i-1/2} \quad 0 < u^n_{i+1/2} - a^n_{i+1/2}$, one gets:

$$\begin{bmatrix} 0 \\ l^{(2)}_{i-1/2} \\ l^{(3)}_{i-1/2} \end{bmatrix} \cdot \left(\frac{w_i^{n+1} - w_i^n}{\Delta t} + \frac{f_i^n - f_{i-1}^n}{\Delta x} \right) + \begin{bmatrix} l^{(1)}_{i} \\ 0 \\ 0 \end{bmatrix}$$

$$\cdot \left[\frac{w_i^{n+1} - w_i^n}{\Delta t} + \frac{\lambda_i^{(1)} \dfrac{w_{i+1}^n - w_{i-1}^n}{2\,\Delta x}}{1 + \dfrac{\Delta t}{\Delta x} (\lambda^{(1)}_{i+1/2} - \lambda^{(1)}_{i-1/2})} \right] = 0.$$

Finally, at a 'shock' point, four or more characteristics converge. A conservative scheme consists in combining the box equations coming from the left and right boxes as:

$$\begin{bmatrix} \tilde{l}_i^{(1)} \\ \tilde{l}_i^{(2)} \\ \tilde{l}_i^{(3)} \end{bmatrix} \cdot \left(\frac{w_i^{n+1} - w_i^n}{\Delta t} + \frac{f_{i+1}^n - f_i^n}{\Delta x} + \frac{f_i^n - f_{i-1}^n}{\Delta x} \right) = 0.$$

Note that, for a characteristic $\lambda^{(j)}$, which changes sign, the scheme is analogous to the above mixed scheme, in particular as concerns the shock and sonic point operators. Hence, this scheme computes the correct shock speed, and prevents expansion shocks by forcing a smooth transition at sonic condition. The time derivatives are obtained from the solution of a 3×3 system whose matrix is composed of the left eigenvectors. At supersonic points and shock points, the matrix of left eigenvectors can be replaced by the unit matrix.

5. USE OF DISCRETE EINGENVALUES AND EIGENVECTORS

The above scheme can be written in a different, but equivalent form, by making use of the property of the Roe averages. This will allow for an easier interpretation of the scheme. We have the discrete identities:

$$l_{i-1/2}^{(j)} \cdot \frac{f_i^n - f_{i-1}^n}{\Delta x} = \lambda_{i-1/2}^{(j)} \, l_{i-1/2}^{(j)} \cdot \frac{w_i^n - w_{i-1}^n}{\Delta x},$$

$$l_{i+1/2}^{(j)} \cdot \frac{f_{i+1}^n - f_i^n}{\Delta x} = \lambda_{i+1/2}^{(j)} \, l_{i+1/2}^{(j)} \cdot \frac{w_{i+1}^n - w_i^n}{\Delta x}.$$

For the sonic point operator, the eigenvalues and left eigenvectors are evaluated at the nodal state w_i^n. This insures that the scheme is not in conservation form.

At the shock point, there are four or more characteristics flowing to point i. The situation is identical to that of the Burgers model and is treated identically, by adding the space derivatives from the two surrounding boxes, thus producing a non-consistent but conservative scheme. However, choices for the eigenvectors, denoted $\tilde{l}_i^{(j)}$ in the above formula, need to be made. It is proposed to carry the analogy with the B-H equation one step further, so that the Euler equations can be linearized in a similar manner as was done for the simpler model, and if two characteristics of the same family converge, two wave speeds of opposite signs will be present as fundamental feature. For example, if $\lambda_{i-1/2}^{(1)} > 0$ and $\lambda_{i+1/2}^{(1)} < 0$, the equation for the $CR^{(1)}$ compatibility relation can be rewritten as:

$$\tilde{l}_i^{(1)} \cdot \left(\frac{w_i^{n+1} - w_i^n}{\Delta t} + \lambda_{i+1/2}^{(1)} \frac{w_{i+1}^n - w_i^n}{\Delta x} + \lambda_{i-1/2}^{(1)} \frac{w_i^n - w_{i-1}^n}{\Delta x} + (A_{i+1/2} - \lambda_{i+1/2}^{(i)} I) \frac{w_{i+1}^n - w_i^n}{\Delta x} \right.$$
$$\left. + (A_{i-1/2} - \lambda_{i-1/2}^{(1)} I) \frac{w_i^n - w_{i-1}^n}{\Delta x} \right) = 0.$$

One can choose the vector $\tilde{l}_i^{(1)}$ such that the last two terms in the above equation cancel out. The search for such a vector is made easier by looking for the value $\tilde{\theta}$ of the linear combination:

$$\tilde{l}_i^{(1)} = \tilde{\theta} l_{i+1/2}^{(1)} + (1 - \tilde{\theta}) l_{i-1/2}^{(1)}.$$

Using the property of the eigenvectors, the condition reduces to:

$$\tilde{\theta} l^{(1)}_{i+1/2} \cdot (A_{i-1/2} - \lambda^{(1)}_{i-1/2} I) \cdot \frac{w^n_i - w^n_{i-1}}{\Delta x} + (1-\tilde{\theta}) l^{(1)}_{i-1/2} \cdot (A_{i+1/2} - \lambda^{(1)}_{i+1/2} I) \cdot \frac{w^n_{i+1} - w^n_i}{\Delta x} = 0.$$

With this choice, one can rewrite the scheme, say for compatibility relation $CR^{(1)}$, in updating form as:

$$\text{(a) } \lambda^{(1)}_{i-1/2} > 0 \quad l^{(1)}_{i-1/2} \cdot w^{n+1}_i = l^{(1)}_{i-1/2} \cdot \left(w^n_i - \frac{\Delta t}{\Delta x} \lambda^{(1)}_{i-1/2} (w^n_i - w^n_{i-1}) \right)$$

$$\lambda^{(1)}_{i+1/2} < 0 \quad l^{(1)}_{i+1/2} \cdot w^{n+1}_i = l^{(1)}_{i+1/2} \cdot \left(w^n_i - \frac{\Delta t}{\Delta x} \lambda^{(1)}_{i+1/2} (w^n_{i+1} - w^n_i) \right)$$

$$\text{(b) } l^{(1)}_i \cdot w^{n+1}_i = l^{(1)}_i \cdot \left[w^n_i - \frac{\dfrac{\Delta t}{2\,\Delta x} \lambda^{(1)}_i (w^n_{i+1} - w^n_{i-1})}{1 + \dfrac{\Delta t}{\Delta x} (\lambda^{(1)}_{i+1/2} - \lambda^{(1)}_{i-1/2})} \right]$$

$$\text{(c) } \tilde{l}^{(1)}_i \cdot w^{n+1}_i = \tilde{l}^{(1)}_i \cdot \left(w^n_i - \frac{\Delta t}{\Delta x} \lambda^{(1)}_{i+1/2} (w^n_{i+1} - w^n_i) - \frac{\Delta t}{\Delta x} \lambda^{(1)}_{i-1/2} (w^n_i - w^n_{i-1}) \right)$$

This is similar to the mixed scheme for the B-H equation, hence the stability conditions are identical. Because this is a system, the left eigenvectors can be interpreted as projection operators of a L_∞-stable scheme for the unknown vector w. The scheme has been found in numerical experiments to be very robust. There is no free parameter to adjust. At steady state, the scheme is second-order accurate.

6. SOME RESULTS

The shock tube test case of Sod is simulated on a mesh with 1001 points, at a CFL condition number of one, for 500 steps. The results are presented in Figure 1 and are in good agreement with the exact solution.

The choked flow in a converging–diverging nozzle of equation $g(x) = 1 + 4(x - 0.5)^2$ with an exit pressure $p_{\text{exit}} = \gamma p^*$, yields a compression shock in the diverging part. The steady state

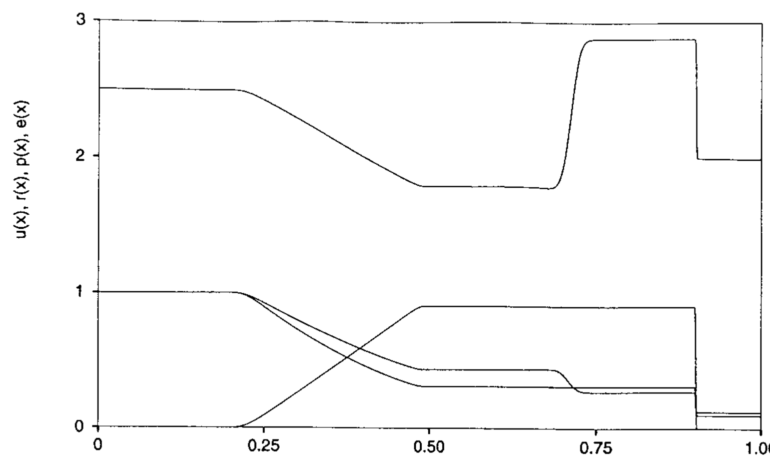


Figure 1. Shock tube—Sod test case. Velocity, density, pressure and specific energy distributions.

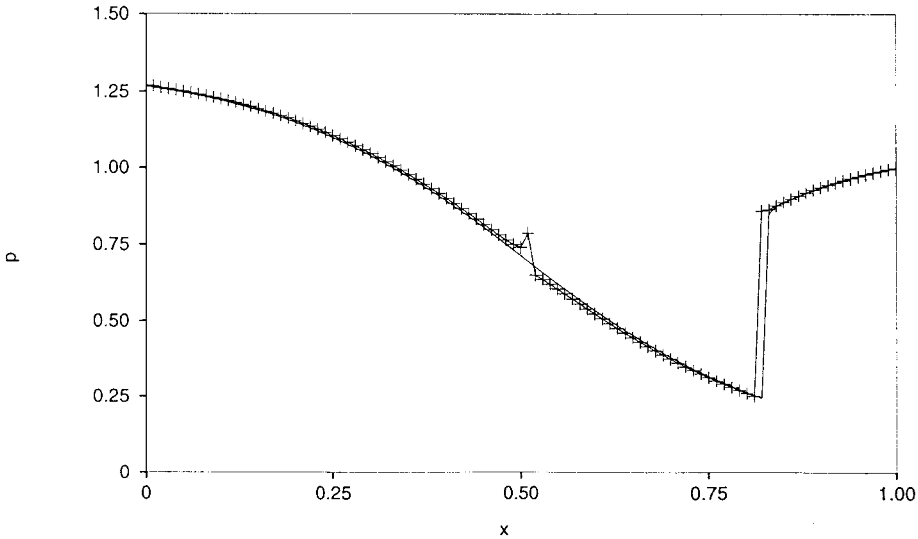


Figure 2. Choked flow in a converging–diverging nozzle. Comparison of the pressure distributions using the Roe scheme and the box-scheme.

solution is compared with Roe scheme. It can be seen in Figure 2, that the Roe scheme, without the artificial viscosity at the sonic point, does not give a smooth transition.

The last example corresponds to the physical problem of the start-up of a supersonic wind tunnel. The high pressure reservoir is connected to the atmospheric exhaust by a double throated duct. The forward converging–diverging nozzle has a unit throat area and reaches a maximum area at the test section $A_{\text{test}} = 1.25$. The converging–diverging diffuser nozzle has a variable throat that depends on a parameter α. As α decreases, the second throat area increases. The equations for the geometry are given by:

$$\begin{cases} g(x) = \dfrac{9}{4}(2x-1)^4 - \dfrac{3}{2}(2x-1)^2 + \dfrac{5}{4}, & 0 \le x \le 0.5 \\ g(x) = \alpha\left\{\dfrac{9}{4}(2x-1)^4 - \dfrac{3}{2}(2x-1)^2\right\} + \dfrac{5}{4}, & 0.5 \le x \le 1, \quad 0 \le \alpha \le 1 \end{cases}.$$

In order for the test section to be shock-free, the first shock must be swallowed by the second throat. This requires the second throat to satisfy the inequality [6]:

$$\frac{A_2}{A_1^*} \ge \frac{A_2^*}{A_1^*}\,[M_{\text{test}}].$$

Here, this corresponds to $A_2/A_1^* \ge 1.1169$.

To simulate the start-up, the initial condition corresponds to air at rest, with the left boundary point ($i = 1$) representing the reservoir condition, the rest of the duct being at atmospheric pressure. In dimensionless form:

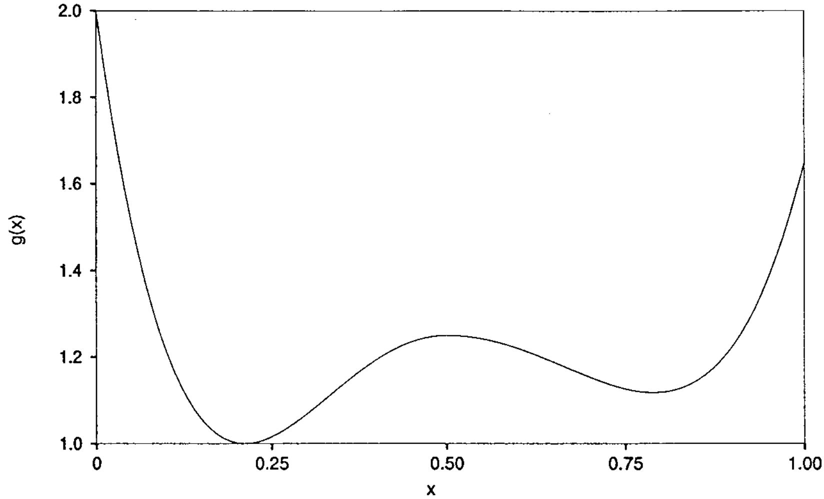


Figure 3. Supersonic wind tunnel area distribution.

$$\begin{cases} u_1 = 0 \\ \rho_1 = \left(\dfrac{\gamma+1}{2}\right)^{1/(\gamma-1)} \\ p_1 = \dfrac{\rho_1^\gamma}{\gamma} \end{cases} \qquad \begin{cases} u_i = 0 \\ \rho_i = \dfrac{2\gamma}{\gamma+1} p_{\text{exit}}, \quad i = 2, \ldots, ix, \\ p_i = p_{\text{exit}} \end{cases}$$

where $p_{\text{exit}} = 0.9$. The mesh has 101 points. The code is run with $A_2/A_1^* = 1.117$ for 30000 iterations with a CFL number one. The wind tunnel geometry is shown in Figure 3. The evolution of pressure with time is plotted in Figure 4 every 1000 iterations. It can be seen that the first shock takes a long time to be 'swallowed' by the second throat. The shock speed

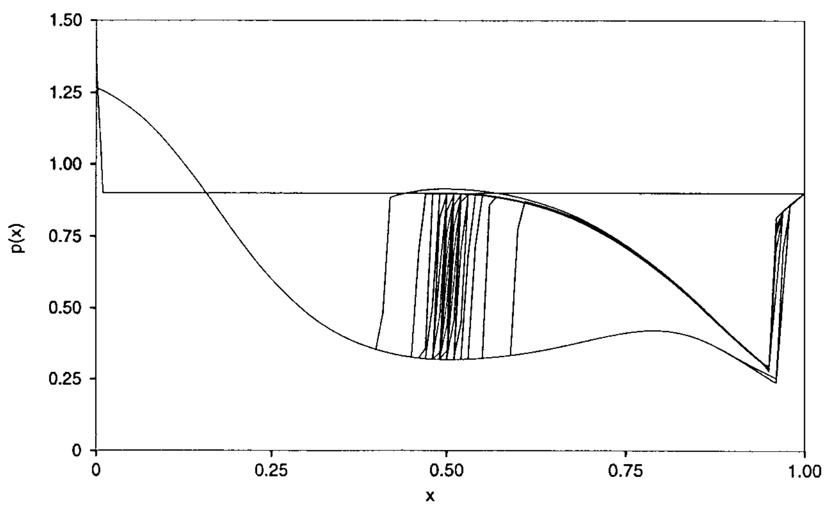


Figure 4. Supersonic wind tunnel start-up. Evolution of pressure with iterations for $A_2/A_1^* = 1.117$.

reaches a minimum just slightly above zero at the test section. Once in the converging part of the diffuser nozzle, the first shock moves quickly to the right of the second throat and coalesces with the second shock to form a single stronger shock as the flow reaches steady state.

7. CONCLUSION

A characteristic based box-scheme for the Euler equations has been presented. It can be interpreted as a natural extension of the Murman–Cole scheme to a system, in which a L_∞-stable scheme for the Euler equation is projected onto the left eigenvectors, to advance the solution in time. The scheme has been applied to many gas dynamics problems and exhibited excellent properties of robustness and accuracy. The scheme is free of tuning parameters. At steady state it is second order accurate. In future work, this scheme could serve as an approximate Riemann solver for the Euler equations in multi-dimensions.

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